Math Background

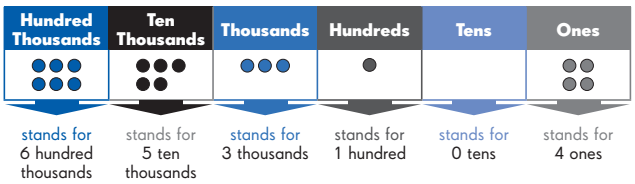
Additional Teaching Support

- Online Transition Guide
- Online Professional Development Videos

The Number System (NS)

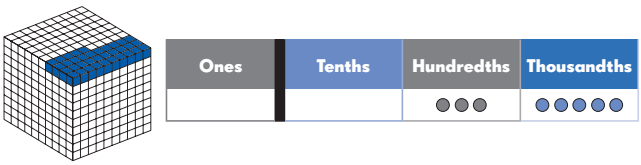
A Variety of Methods for Representing Numbers

Students using the Singapore approach to mathematics become familiar with multiple ways to represent numbers beginning in the primary grades. From their very earliest experiences with whole-number place value, students learn to use place-value chips to show the powers of ten that are the basis of our number system. Here, for example, a student uses place-value chips and a place-value chart to model the number 653,104.

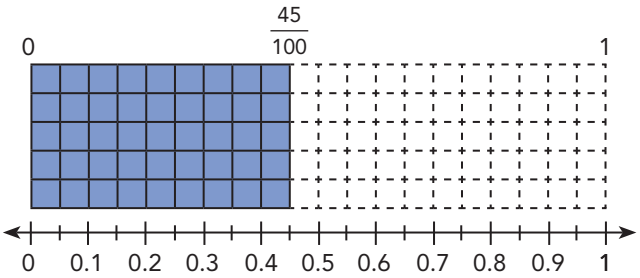


When students broaden their understanding of "number" beyond whole numbers to include fractions in decimal form, previously-learned and familiar models continue to play a crucial role in concept building.

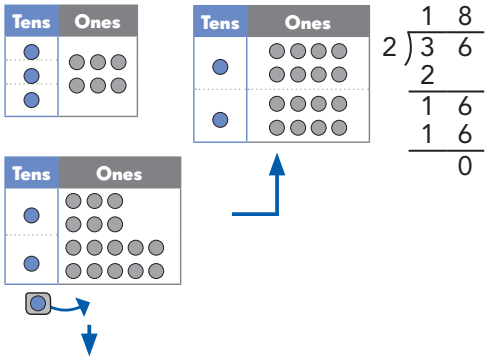
Place-value chips are often juxtaposed with base-ten models to help students interpret numbers in different ways. Here are two different representations for the decimal 0.35.



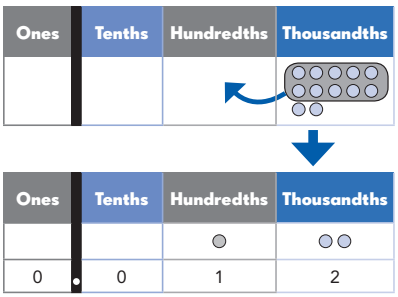
The set of rational numbers includes fractions as well as decimals. But place-value charts are not particularly useful for fraction models. Instead, grid models are combined with number lines to show how fractions and decimals are related. This model helps students relate $\frac{45}{100}$ to 0.45.



The multiple representations for numbers in Singapore math go far beyond developing basic number sense. The same models are used to help students learn how—and why—computational algorithms work. An example of whole-number division from earlier grades illustrates how students can use place-value chips to model the regrouping that is part of almost all number computation.

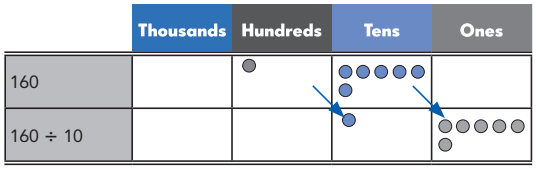


Regrouping is also involved in understanding place value with decimals. Here, students use place-value chips and a place-value chart to show why 12 thousandths is written in the decimal form 0.012.

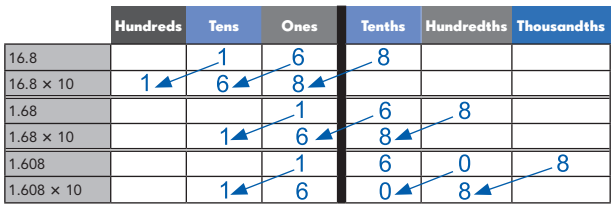


Math Background

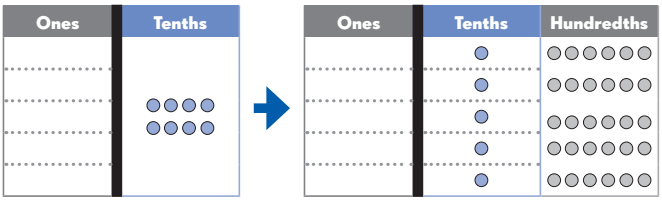
The same manipulatives can be used to model operations. Below, place-value chips and a place-value chart show division of the whole number 160 by 10.



Here, the process is extended to decimals. Multiplying and dividing by powers of 10 plays a key role in the development of decimal concepts.

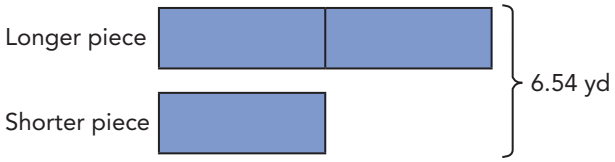


Students can also use place-value chips to model decimal operations that do not involve tens. They go through a series of regroupings until the final layout reveals the answer. This model illustrates the first and last stages of dividing 0.8 by 5.



Students who have studied Singapore math have used bar modeling and the unitary method to solve whole-number word problems. The same models can be used to solve problems involving fractions and decimals, as below.

A piece of metal is 6.54 yards long. It is cut into two pieces. One piece is twice as long as the other. What is the length of the longer piece?



From the model, 3 units = 6.54 yd.

3 units $\rightarrow$ 6.54

1 unit $\rightarrow 6.54 \div 3 = 2.18$

2 units $\rightarrow 2.18 \cdot 2 = 4.36$

The longer piece is 4.36 yards long.

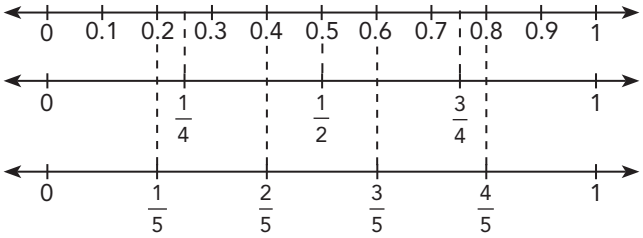
From number meanings to operations to problem solving, the Singapore use of models gives students powerful visualizing tools. The bar modeling and unitary method that work so well with whole numbers are equally effective in problem-solving applications involving fractions and decimals.

Number Line Models for Equivalence and Integers

Number line models emphasize the connections between fractions and decimals and help to lay the groundwork for an eventual understanding of the set of real numbers.

Example 1 Comparing Numbers in Different Forms

Which is greater, $\frac{3}{5}$ or 0.7?



$\frac{3}{5} = 0.6$

0.6 lies to the left of 0.7.

So, $\frac{3}{5} < 0.7$.

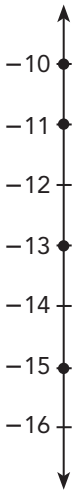
Example 2 Representing Integers

Draw a vertical number line to represent the following set of numbers.

–13, –10, –15, –11

Choose a number less than the least number in the set. The least number is −15, so start with −16.

Draw the number line from the bottom up until you have included all four numbers. Add one more integer at the top.



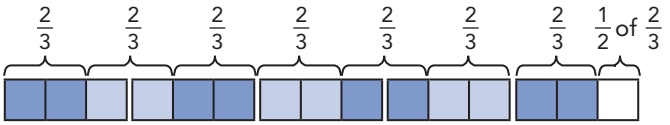
Fraction and Decimal Multiplication and Division

By the end of Grade 5, students have mastered all four operations with whole numbers, as well as addition and subtraction of fractions and decimals. This course concludes that work by introducing multiplication and division with both fractions and decimals. Course 2 will introduce use of the four operations with integers.

Example 3 Dividing by a Fraction

Betty used $\frac{2}{3}$ jar of poster paint to paint one election poster. How many posters can she complete with 5 jars of paint?

To solve the problem, divide 5 by $\frac{2}{3}$. First divide each of 5 whole bars into 3 equal parts. Then shade the parts in groups of $\frac{2}{3}$.



Number of two-thirds in 2 wholes = 3

Number of two-thirds in 1 whole = $1\frac{1}{2}$ or $\frac{3}{2}$

Number of two-thirds in 5 wholes = $5 \cdot \frac{3}{2}$

So, $5 \div \frac{2}{3} = 5 \cdot \frac{3}{2}$

$= \frac{15}{2}$

$= 7\frac{1}{2}$

Betty can paint $7\frac{1}{2}$ posters with 5 jars of paint.

Bar Modeling with Fractions and Decimals

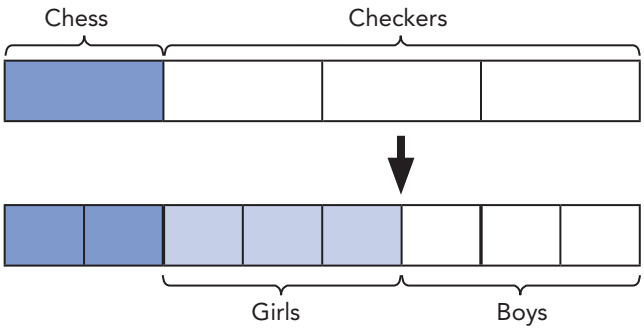
Because educators in the United States may be unfamiliar with the unitary method and bar modeling, these final two examples illustrate the power of these visualization strategies.

Example 4 Unitary Models with Fractions

Derrick's school has a chess and checkers club.

$\frac{1}{4}$ of the members play in chess tournaments.

The rest compete in checkers. One-half of the checkers players are girls. The club has 72 members in all. How many checkers players are boys?



Students begin by drawing a bar divided into equal fourths to show the fractions of the club that play chess and checkers.

3 of the 4 parts of the bar show checker players. Because half of the checkers players are girls, those 3 bars must be divided into 2 equal parts. To do that, students must divide the 3 equal parts into 6 equal parts. So, the whole bar must be divided into 8 equal parts.

The model shows that $\frac{3}{8}$ of the members of the club who play checkers are boys. The whole bar represents the 72 members in the club.

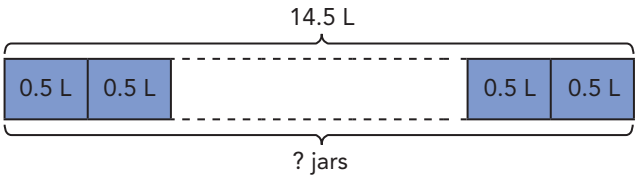
8 units $\rightarrow$ 72
1 unit $\rightarrow$ $72 \div 8 = 9$
3 units $\rightarrow$ $3 \cdot 9 = 27$

There are 27 boys in the club who play checkers.

Commentary The method of finding the value of one of several equal units is called the unitary method. In this example, students use the method to solve a complex problem involving fractions. They will also find this approach effective in problems that involve ratio, proportion, and percent.

Example 5 Modeling Decimal Division

Gayle and her brother made 14.5 L of tomato salsa. They are filling gift jars that each hold 0.5 L. How many jars can they fill?



$14.5 \div 0.5 = 29$

Gayle and her brother can fill 29 jars with tomato salsa.

Commentary This particular division model may be new to most teachers and students. The dashed lines at the center of the bar indicate that the quantity of equal parts the whole bar is divided into is unknown. Because the model works equally well with decimals and fractions as with whole numbers, students can make use of it for any type of division situation.

Additional Teaching Resource

For additional reading, see *The Singapore Model Method for Learning Mathematics* published by the Ministry of Education of Singapore and *Bar Modeling: A Problem-solving Tool* by Yeap Ban Har, published by Marshall Cavendish Education.